Moment of a Force About a Point

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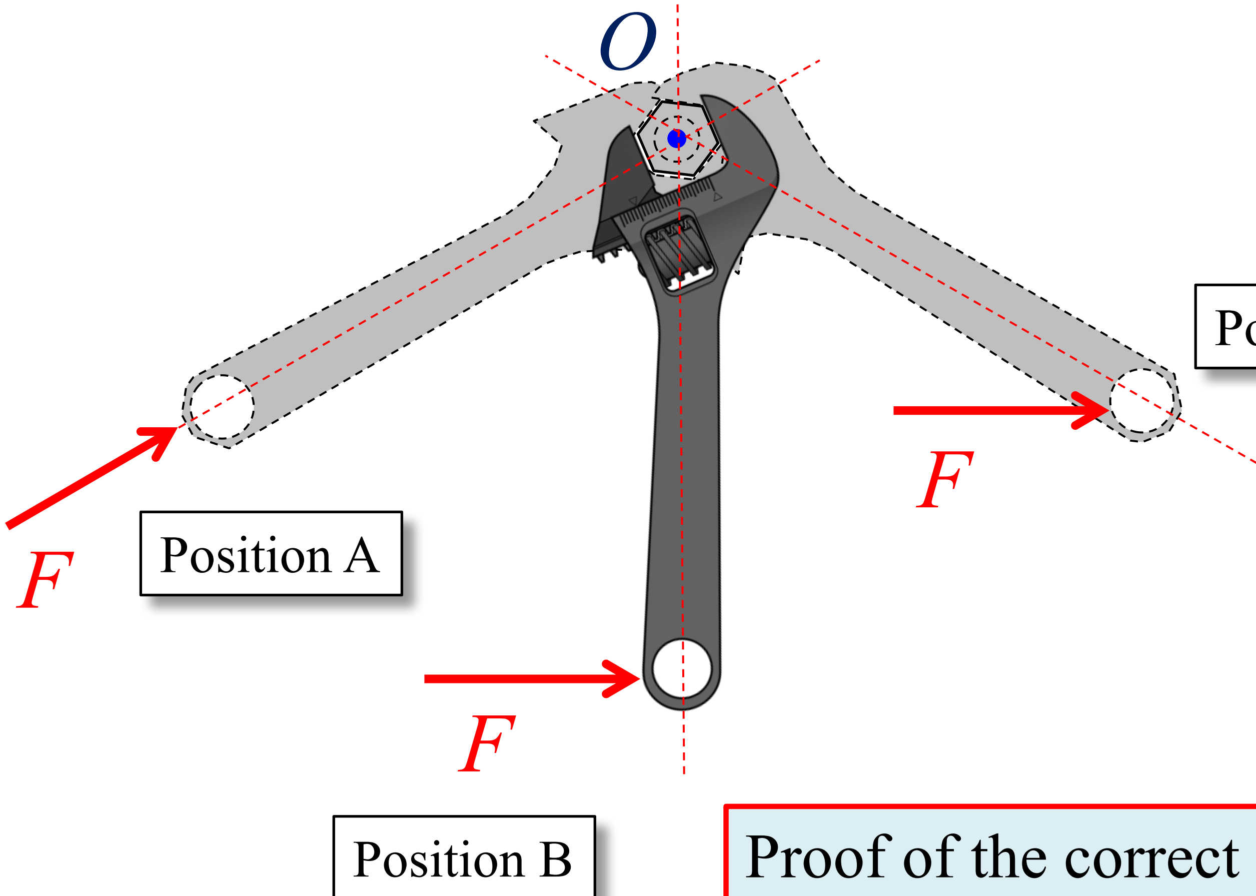
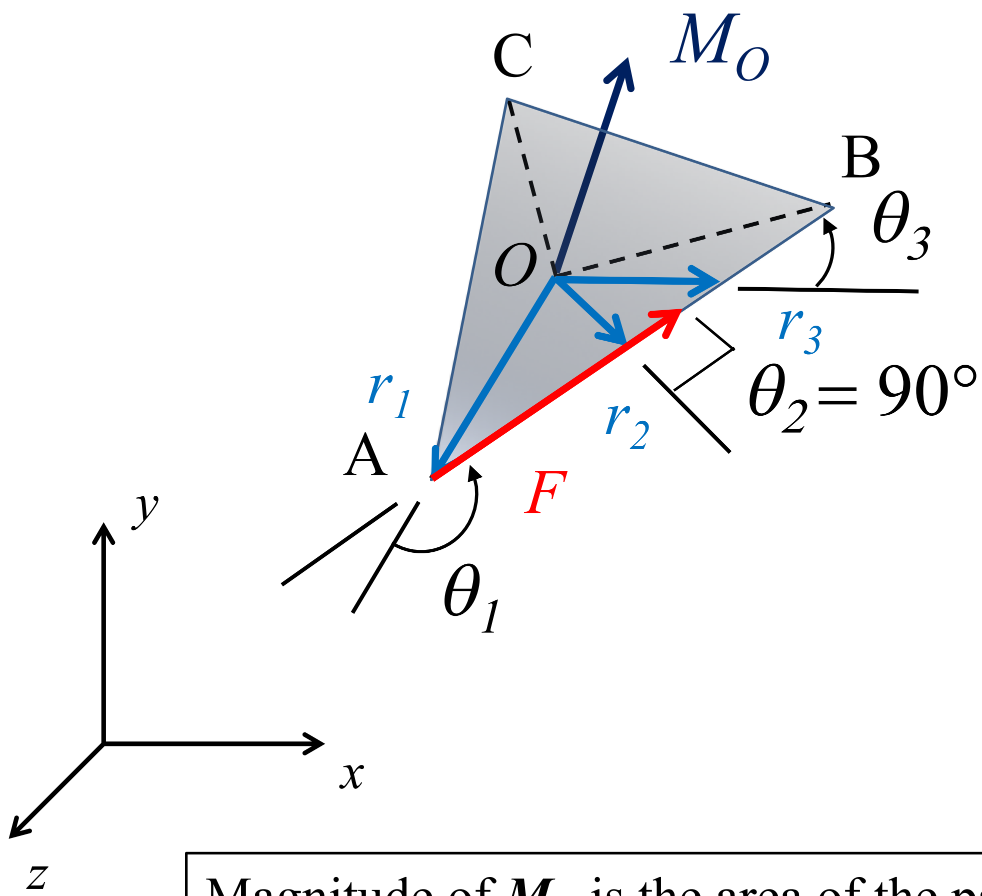
Which application of the force *F* would provide the most rotation to loosen the nut at point *O*?



Proof of the correct answer lies in the concept of the moment of a force about a point

Moment of a Force *F* about a Point *O*



$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

$$M_O = rF \sin\theta$$

r is a position vector that must satisfy:

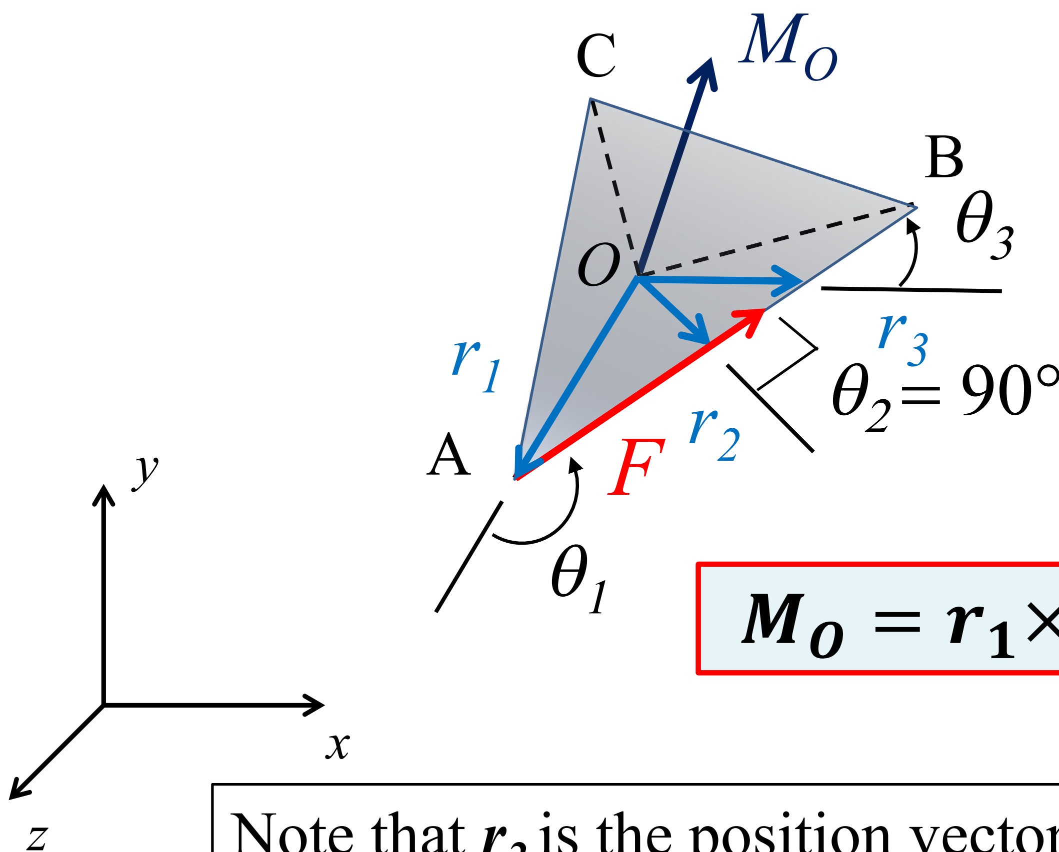
- Tail of ***r*** is at point ***O***;
- Tip can be on any point on the line-of-action of ***F***

Magnitude of $\boldsymbol{M_O}$ is the area of the parallelogram defined by ***r*** and ***F***

Direction of $\boldsymbol{M_O}$ is perpendicular to the plane defined by ***r*** and ***F***

Sense of $\boldsymbol{M_O}$ is defined by the right-hand rule

Moment of a Force *F* about Point *O*



$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

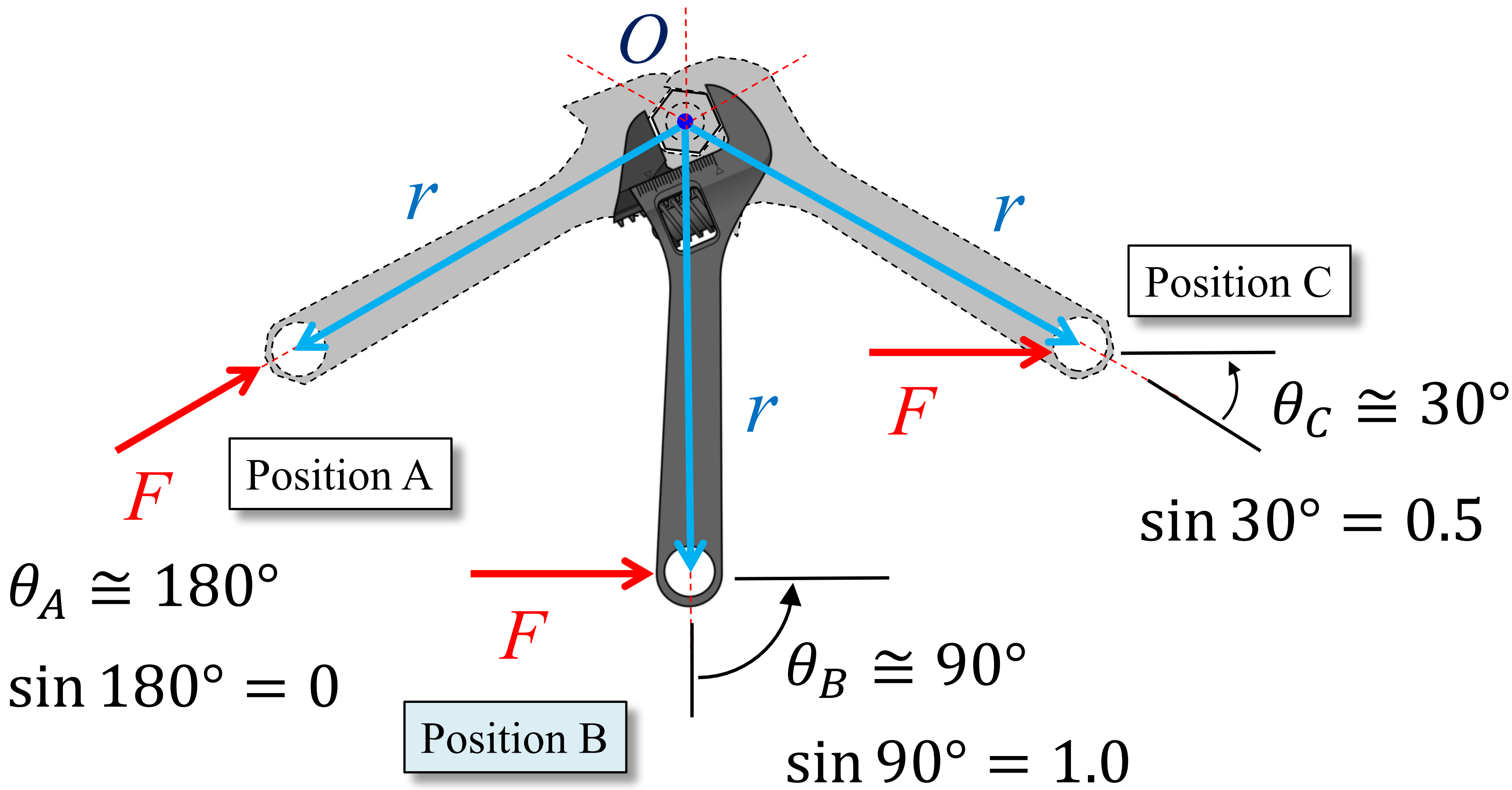
$$M_O = rF \sin\theta$$

$$\boldsymbol{M_O} = \boldsymbol{r_1} \times \boldsymbol{F} = \boldsymbol{r_2} \times \boldsymbol{F} = \boldsymbol{r_3} \times \boldsymbol{F}$$

Note that $\boldsymbol{r_2}$ is the position vector perpendicular to the line-of-action of ***F***. The length of this perpendicular position vector is usually denoted as *d*

$$M_O = r_2 F \sin 90° = dF$$

Let's Examine Our Initial Question Applying the Concept of Moment of a Force About a Point

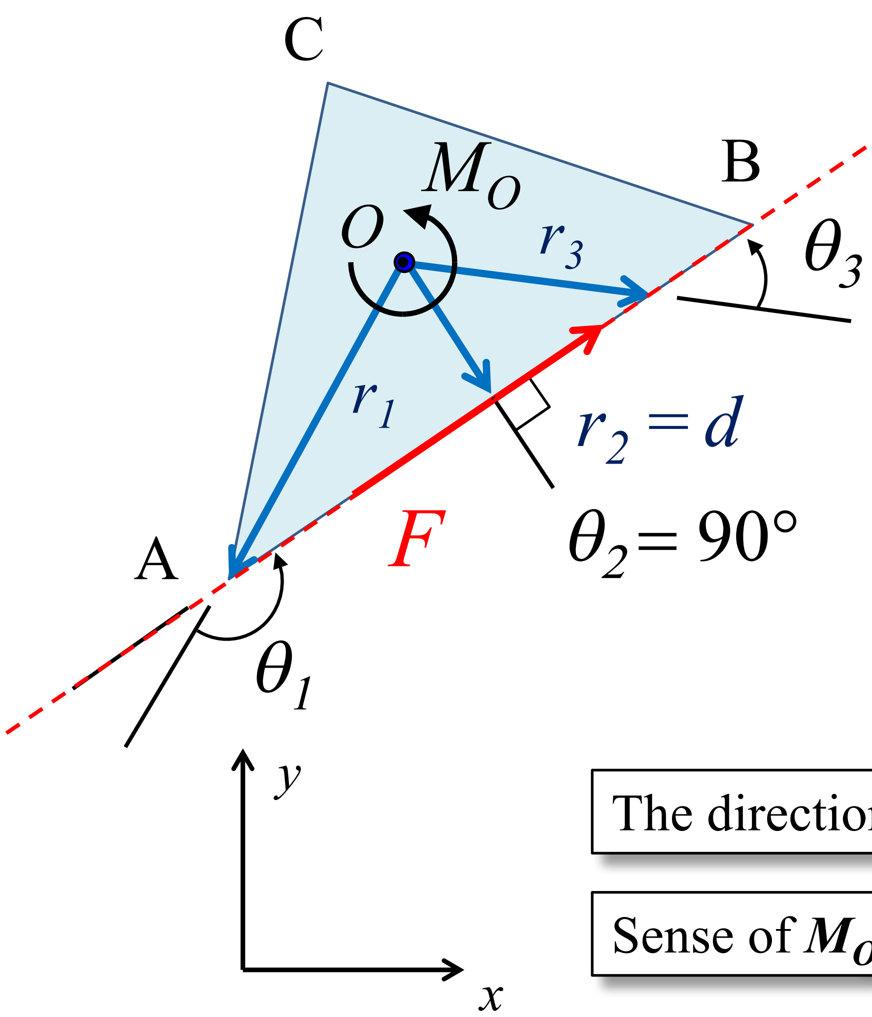


$M_O^A \cong rF(0) \cong 0$

$M_O^B \cong rF(1.0) \cong rF$

$M_O^C \cong rF(0.5) \cong 0.5rF$

Moment of a Force about a Point for Planar Problems



$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

$$M_O = rF \sin\theta$$

$$M_O = dF$$

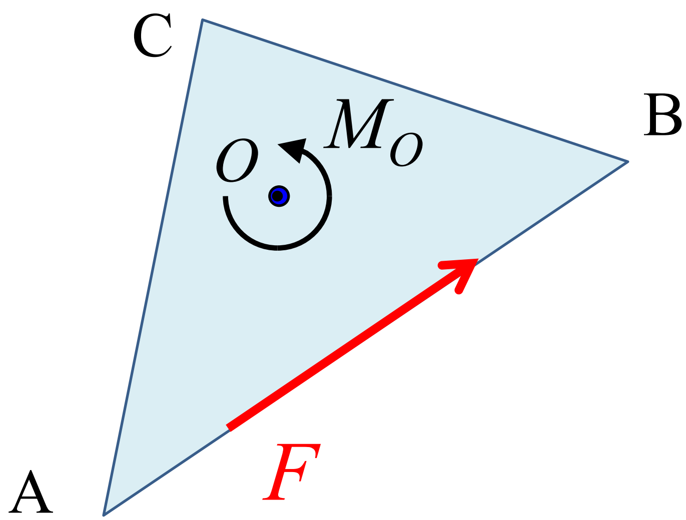
r is a position vector that must satisfy:

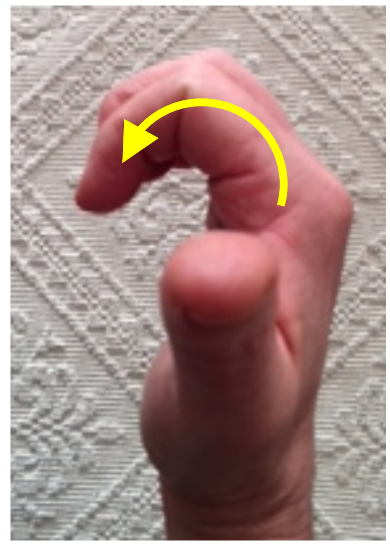
- Tail of ***r*** is at point ***O***;
- Tip can be on any point on the line-of-action of ***F***

The direction of $\boldsymbol{M_O}$ will always be in the z direction

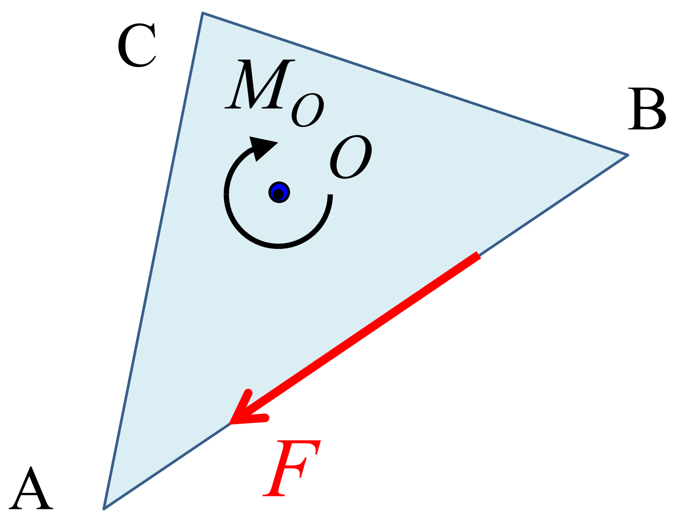
Sense of $\boldsymbol{M_O}$ is defined by the right-hand rule

Sense of Moment for Planar Problems





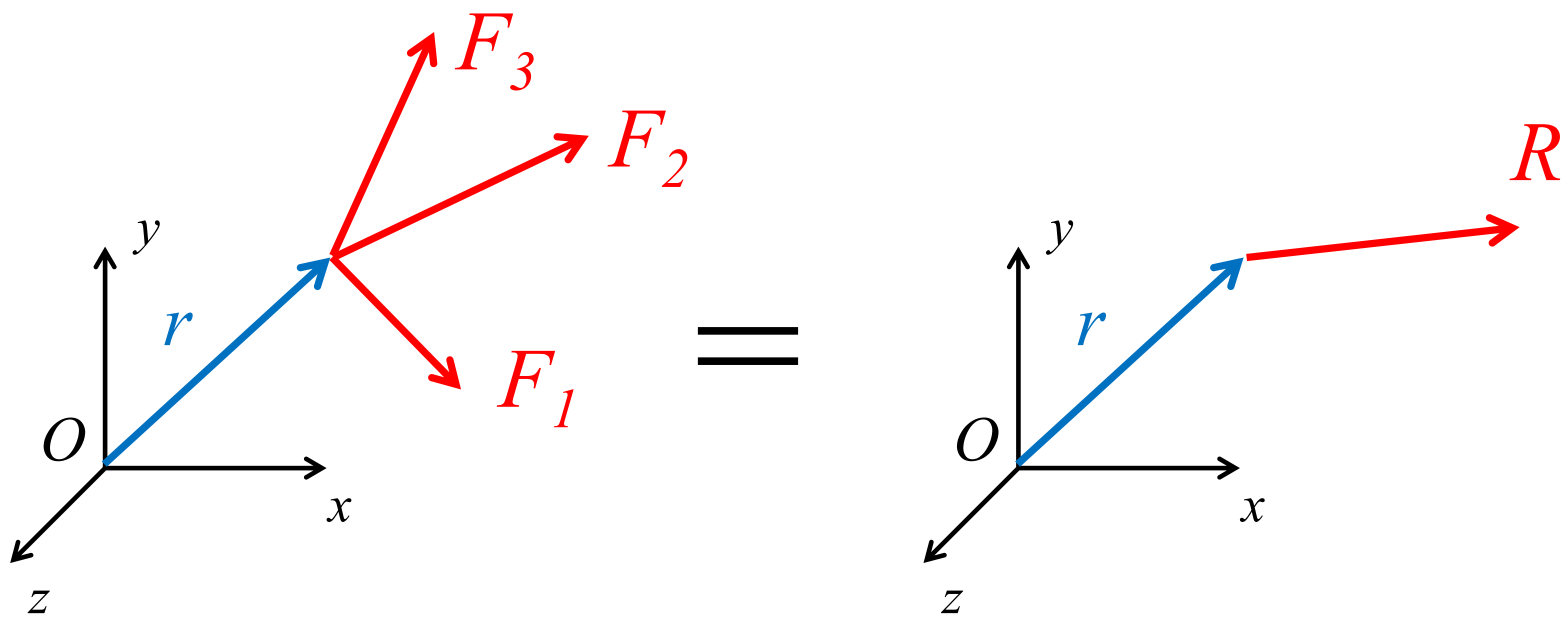
The direction of $\boldsymbol{M_O}$ will always be in the z direction for a planar problem



The sense of $\boldsymbol{M_O}$ is defined by the right-hand rule

- Counter-clockwise (positive *z* direction)
- Clockwise (negative *z* direction)

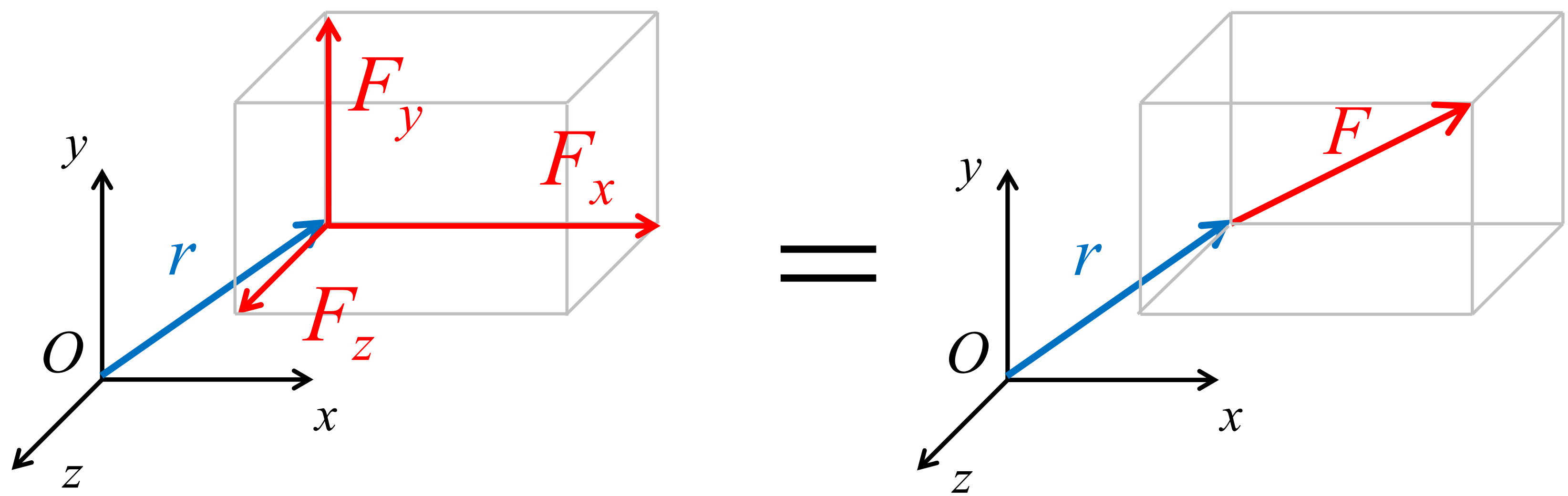
Varignon's Theorem



$$M_O = r \times F_1 + r \times F_2 + r \times F_3$$ = $$M_O = r \times (F_1 + F_2 + F_3) = r \times R$$

The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O

Moment of a Force in Cartesian Vector Form about a Point



$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F_x}\hat{\imath} + \boldsymbol{r} \times \boldsymbol{F_y}\hat{\jmath} + \boldsymbol{r} \times \boldsymbol{F_3}\hat{\boldsymbol{k}} \quad = \quad \boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

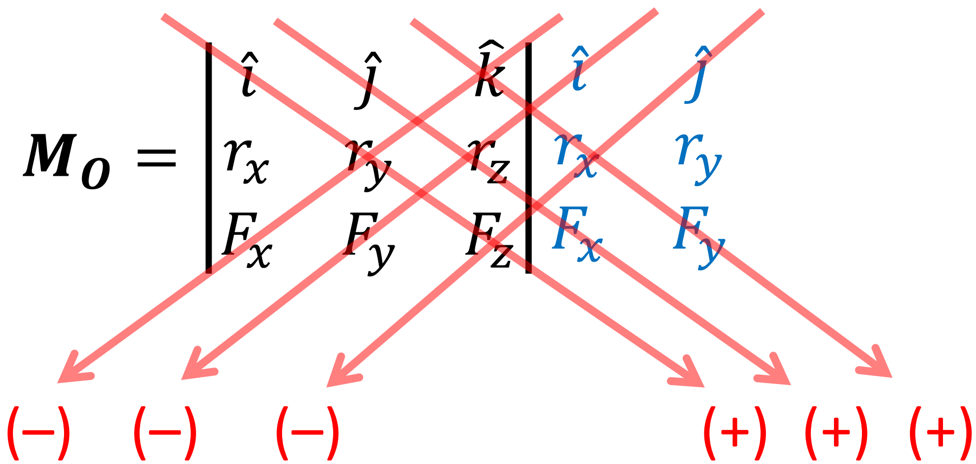
Moment of a Force about a Point when the Position Vector and Force Vector are in Cartesian Vector Form

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$$

$$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

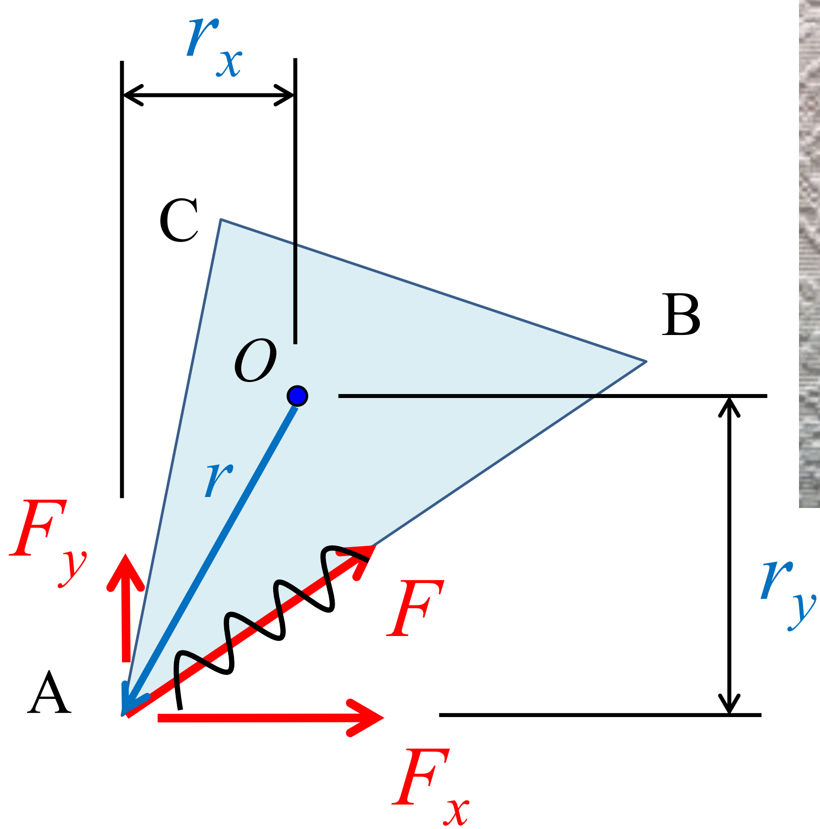
$$\boldsymbol{M_O} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

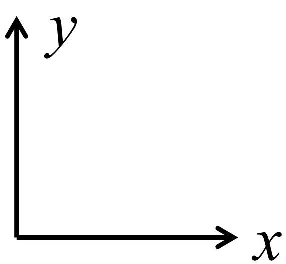


Almost always the best way to calculate the moment of a force about a point for three-dimensional problems

$$\boldsymbol{M_O} = (r_yF_z - r_zF_y)\hat{\imath} + (r_zF_x - r_xF_z)\hat{\jmath} + (r_xF_y - r_yF_x)\hat{k}$$

Moment of a Force about a Point for Planar Problems





$$M_O = +F_x r_y - F_y r_x$$

Calculate the moment of each component of ***F*** using the perpendicular distance from point ***O***.

Add the moment of each component (counter-clockwise rotation is positive and clockwise rotation is negative) to find the moment of the force ***F*** about point ***O***.